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In Accordance with a "More Majestic Order"

The New Math and the Nature of Mathematics at Midcentury

*By Christopher J. Phillips**

ABSTRACT

The "new math" curriculum, one version of which was developed in the 1950s and 1960s by the School Mathematics Study Group under the auspices of the National Science Foundation, occasioned a great deal of controversy among mathematicians. Well before its rejection by parents and teachers, some mathematicians were vocal critics, decrying the new curriculum because of the way it described the practice and history of the discipline. The nature of mathematics, despite the field's triumphs in helping to win World War II and its midcentury promotion as the key to a modern technological society, was surprisingly contested in this period. Supporters of the School Mathematics Study Group, like its director, Edward Begle, emphasized the importance of portraying mathematics as a system of abstract structures, while opponents like Morris Kline argued that math was essentially a tool for understanding the natural world. The debate about the curriculum—and the role of mathematicians in its design—was also a debate about the underlying identity of the subject itself.

IN ONE OF THE PIVOTAL SCENES of George Orwell's dystopian novel *Nineteen Eighty-Four*, the protagonist, Winston, faces his tormenter, the state agent O'Brien:

> "Do you remember," [O'Brien] went on, "writing in your diary, 'Freedom is the freedom to say that two plus two make four'?"

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> "Yes," said Winston.
> O'Brien held up his left hand, its back towards Winston, with the thumb hidden and the four fingers extended.
> "How many fingers am I holding up, Winston?"
> "Four."
> "And if the Party says that it is not four but five—then how many?"
> "Four."
> The word ended in a gasp of pain.

Winston would eventually signify his surrender to "Big Brother" by tracing "2+2=5" into the dust on a table.[1]

Only a few years after Orwell's novel was published, Arthur Bestor, a historian at the University of Illinois, would quote from it as part of his argument that American citizens needed more disciplined intelligence. Bestor's 1953 screed *Educational Wastelands* was a widely read critique based on the claim that American public schools had abandoned the scholarly disciplines in favor of what he considered trendy bromides like life-adjustment education. Bestor feared that undermining the disciplines, as Orwell had intimated, would place the entire nature of democratic society at risk.[2]

For many educational critics, promotion of the traditional academic disciplines was of increasing importance during the early Cold War. Bestor, for example, cited the danger of McCarthyism and "Red Scares" as evidence for the importance of disciplined, liberal education. Anti-Communists had conflicting ideas about how to counteract Soviet influence, but there was broad agreement that the promotion of disciplined intelligence was critical not only for checking the demagoguery of Joseph McCarthy's followers but also for guarding against any actual Communist threat. Fears of anti-intellectualism and American stagnation in the competition with the Soviet Union drove many to think deeply about the relationships among education, "Western" democratic values, social order, and complex domestic and international security challenges. The Soviet launch of *Sputnik* in October 1957 seemed to bolster the view of critics like Admiral H. G. Rickover, who had long claimed that the lack of intellectual discipline in schools was ultimately a matter of national security. Just after the launch, Arkansas Senator William Fulbright indeed reiterated that "the heart of the contest with the Soviet Union is education." The promotion of disciplined intelligence, though, usually meant little more than the promotion of intellectual habits characteristic of university professors. As the historian David Hollinger has noted, academic science was a "magnificent ideological resource" for intellectuals seeking to define the nature—and relative value—of public or private knowledge. Mid-century educational reformers claimed that more rigorous scientific education would not only solve the claimed "scientific manpower" shortage but also promote intellectual rigor: academic critics responded to early Cold War exigencies by proposing their own disciplinary methods as models for rational thought generally.[3]

[1] George Orwell, *Nineteen Eighty-Four* (1949), ed. Bernard Crick (Oxford: Clarendon, 1984), pp. 226, 374–375.

[2] Arthur Bestor, *Educational Wastelands: The Retreat from Learning in Our Public Schools* (1953), 2nd ed. (Urbana: Univ. Illinois Press, 1985), esp. pp. 24, 59. For educational critics and reforms in the 1950s generally see Andrew Hartman, *Education and the Cold War: The Battle for the American School* (New York: Palgrave Macmillan, 2008); John Rudolph, *Scientists in the Classroom: The Cold War Reconstruction of Science Education* (New York: Palgrave, 2002), esp. Ch. 1; and Herbert M. Kliebard, *The Struggle for the American Curriculum, 1893–1958* (Boston: Routledge & Kegan Paul, 1986).

[3] Bestor, *Educational Wastelands*, pp. 179–187; H. G. Rickover, *Education and Freedom* (New York: Dutton, 1959), esp. pp. 35, 38; U.S. Congress, Senate, 85th Cong., 2nd sess., *Congressional Record*, Vol. 104, Pt. 1 (23

Although Orwell had positioned mathematics in this vein—knowing how to add counted as being able to reason freely—midcentury mathematicians themselves did not agree about the nature of their discipline. There was substantial debate about both the conceptual foundations of mathematical practices and the field's historical trajectory. At a time of great prominence and promise for mathematical methods after their wartime successes, mathematicians were hardly unified about what it meant to think mathematically.[4]

In this essay I analyze this debate by using a seemingly oblique approach: mathematicians' work on the midcentury school curriculum. Historians of science have increasingly shown how pedagogy cultivates the material and epistemic practices that come to define what it means to be scientific. At least since Thomas Kuhn, historians have known that textbooks codify the facts, techniques, rules, and exemplars that define the field.[5] These codifications are, in fact, arguments—and they are taken as such by those practitioners who reject the description so offered. In the case of the new math, a federally funded effort to reshape the nation's textbooks between the late 1950s and the mid 1970s, the earliest and some of the most vehement critics were mathematicians who argued that its textbooks misrepresented the "true" nature of mathematical knowledge. The debates

Jan. 1958), p. 872 (Fulbright); and David A. Hollinger, "Science as Weapon in *Kulturkämpfe* in the United States During and After World War II," in *Science, Jews, and Secular Culture: Studies in Mid-Twentieth-Century American Intellectual History* (Princeton, N.J.: Princeton Univ. Press, 1996), pp. 155–174, on p. 160. On education and the many responses to Communism see Hartman, *Education and the Cold War*; Rudolph, *Scientists in the Classroom*; Adam Benjamin Golub, "Into the Blackboard Jungle: Educational Debate and Cultural Change in 1950s America" (Ph.D. diss., Univ. Texas at Austin, 2004), Ch. 2; Ellen Schrecker, *No Ivory Tower: McCarthyism and the Universities* (New York: Oxford Univ. Press, 1986); Michael Kimmage, *The Conservative Turn: Lionel Trilling, Whittaker Chambers, and the Lessons of Anti-Communism* (Cambridge, Mass.: Harvard Univ. Press, 2009); and Richard H. Pells, *The Liberal Mind in a Conservative Age: American Intellectuals in the 1940s and 1950s* (New York: Harper & Row, 1985), esp. Ch. 5. For scientists and the Cold War see, e.g., Jessica Wang, *American Science in an Age of Anxiety: Scientists, Anticommunism, and the Cold War* (Chapel Hill: Univ. North Carolina Press, 1999); Charles Thorpe, *Oppenheimer: The Tragic Intellect* (Chicago: Univ. Chicago Press, 2006), Ch. 7; Ron Robin, *The Making of the Cold War Enemy* (Princeton, N.J.: Princeton Univ. Press, 2001); Jamie Cohen-Cole, *The Open Mind: Cold War Politics and the Sciences of Human Nature* (Chicago: Univ. Chicago Press, 2014); Cohen-Cole, "The Creative American: Cold War Salons, Social Science, and the Cure for Modern Society," *Isis*, 2009, *100*:219–262; Cohen-Cole, "The Reflexivity of Cognitive Science: The Scientist as Model of Human Nature," *History of the Human Sciences*, 2005, *18*:107–139; David Paul Haney, *The Americanization of Social Science: Intellectuals and Public Responsibility in the Postwar United States* (Philadelphia: Temple Univ. Press, 2008), esp. pp. 95–117; Steven Shapin, *The Scientific Life: A Moral History of a Late Modern Vocation* (Chicago: Univ. Chicago Press, 2008), esp. Ch. 4; and Ellen Herman, *The Romance of American Psychology: Political Culture in the Age of Experts* (Berkeley: Univ. California Press, 1995), esp. Ch. 3. On manpower see David Kaiser, "Cold War Requisitions, Scientific Manpower, and the Production of American Physicists after World War II," *Historical Studies in the Physical Sciences*, 2002, *33*:131–159.

[4] Amy Dahan Dalmedico has also explored these issues through locating specific developments in applied mathematics within institutional settings. See Amy Dahan Dalmedico, "An Image Conflict in Mathematics after 1945," in *Changing Images in Mathematics: From the French Revolution to the New Millennium*, ed. Umberto Bottazini and Dahan Dalmedico (New York: Taylor & Francis, 2001), pp. 223–253; and Dahan Dalmedico, "L'essor des mathématiques appliquées aux États-Unis," *Revue d'Histoire des Mathématiques*, 1996, *2*:149–213.

[5] This claim continues to be supported by exemplary studies—most of which, however, focus on college-level textbooks. See, e.g., a recent *Isis* Focus section edited by Marga Vicedo, "Textbooks in the Sciences," *Isis*, 2012, *103*:83–138; Adam R. Shapiro, *Trying Biology: The Scopes Trial, Textbooks, and the Antievolution Movement in American Schools* (Chicago: Univ. Chicago Press, 2013); Adam R. Nelson and John L. Rudolph, eds., *Education and the Culture of Print in Modern America* (Madison: Univ. Wisconsin Press, 2010); Bernadette Bensaude-Vincent, "Textbooks on the Map of Science Studies," *Science and Education*, 2006, *15*:667–670; Kathryn Olesko, "Science Pedagogy as a Category of Historical Analysis: Past, Present, and Future," *ibid.*, pp. 863–880; David Kaiser, ed., *Pedagogy and the Practice of Science: Historical and Contemporary Perspectives* (Cambridge, Mass.: MIT Press, 2005); and Anders Lundgren and Bensaude-Vincent, eds., *Communicating Chemistry: Textbooks and Their Audiences, 1789–1939* (Canton, Mass.: Science History Publications, 2000).

among mathematicians about the virtues of the curriculum grounded and revealed the divisions among mathematicians about the nature of their field.

The vast majority of mathematicians' formal curriculum efforts were concentrated in one program, the National Science Foundation (NSF)–funded School Mathematics Study Group (SMSG).[6] Headed by the Yale (and later Stanford) mathematician Edward Begle, SMSG produced the most important and far reaching of the new math curricula. Begle's group held writing sessions each summer for nearly a decade in a massive effort to design model textbooks appropriate for every student in the country, regardless of grade level or ability. Begle designed the working groups and writing sessions to feature equal numbers of research mathematicians and schoolteachers, hoping that their different areas of expertise would result in mathematically and pedagogically sound textbooks. With over four hundred mathematicians and teachers from thirty-five states participating in SMSG, the group's textbooks—and the debates they prompted—were neither esoteric nor inconsequential. At a cost exceeding $10 million (well over $75 million in today's money), SMSG produced over four million copies of nearly thirty different textbooks and inspired commercial publishers to produce many millions more. One (perhaps overly optimistic) estimate suggested that, at the movement's peak, nearly 75 percent of the nation's high school students and 40 percent of elementary school students were using the new math.[7]

Although it was the only program so closely aligned with the mathematical establishment, SMSG was just one of many curricular reform efforts at midcentury. The American school system's decentralization and its tradition of local control ensured that no single reform dictated what students learned or how instructors taught. Nevertheless, when Congress began to pressure the NSF into expanding its development of precollegiate curricula in the mid 1950s—one of the key initiatives to address the claimed lack of intellectual discipline in the schools—the NSF settled on the compromise of producing model textbooks that would be enticing to schools and teachers, with the goal of eventually influencing commercial publishers. The first of the federal curriculum initiatives was the Physical Sciences Study Committee, headed by the MIT professor Jerrold Zacharias, which established the model for SMSG.[8] Other nongovernmental curriculum reformers had organized in schools of education and had also been working to design new curricula, although never with as much success as SMSG. Secondary school math teachers themselves had been advocating for curricular reform for years, but only after mathematicians agreed to head the project did the efforts receive any substantial federal funding.[9]

[6] For the history of SMSG, in particular, see Christopher J. Phillips, *The New Math: A Political History* (Chicago: Univ. Chicago Press, 2014); Robert W. Hayden, "A History of the 'New Math' Movement in the United States" (Ph.D. diss., Iowa State Univ., 1981); Angela Lynn Evans Walmsley, *A History of the "New Mathematics" Movement and Its Relationship with Current Mathematical Reform* (Lanham, Md.: Univ. Press America, 2003); and William Wooton, *SMSG: The Making of a Curriculum* (New Haven, Conn.: Yale Univ. Press, 1965).

[7] H. Victor Crespy, "A Study of Curriculum Development in School Mathematics by National Groups, 1950–1966: Selected Programs" (Ph.D. diss., Temple Univ., 1969), p. 134; "List of Participants in the Work of SMSG, October 1966," Folder "List of Participants," Box 13.5/86-28/68, School Mathematics Study Group Records, 1958–1977 (hereafter **SMSGR**), Archives of American Mathematics, Dolph Briscoe Center for American History, University of Texas at Austin; and Harry Schwartz, "The New Math Is Replacing Third 'R'," *New York Times*, 25 Jan. 1965, p. 18 (the estimate quoted Howard Fehr, president of the National Council of Teachers of Mathematics).

[8] John Rudolph, "From World War to Woods Hole: The Impact of Wartime Research Models on Curriculum Reform," *Teachers College Record*, 2002, *104*:212–241; and Rudolph, *Scientists in the Classroom* (cit. n. 2).

[9] For the range of reform efforts see Paul C. Rosenbloom, ed., *Modern Viewpoints in the Curriculum: National Conference on Curriculum Experimentation, September 25–28, 1961* (New York: McGraw-Hill, 1964); and Crespy, "Study of Curriculum Development in School Mathematics by National Groups, 1950–1966" (cit. n. 7).

Unlike other contemporary math curriculum projects—for example, the pedagogically innovative University of Illinois Committee on School Mathematics—SMSG was widely understood by teachers, parents, and concerned citizens to represent the "official" version of what math meant to professional mathematicians.[10] Indeed, SMSG was a federal initiative, lavishly funded by taxpayers and run by academic mathematicians through the sponsorship of the American Mathematical Society, the Mathematical Association of America, and the National Council of Teachers of Mathematics. One additional key difference between SMSG and other curriculum reform efforts was the project's sheer scope: SMSG produced at least one set of course materials for every grade and ability level, and Begle kept the working groups awkwardly large in an effort to ensure national scope and influence.

While new curricula rarely satisfy everyone, critiques of SMSG's textbooks were particularly contentious. SMSG's mathematicians were operating on the assumption that developments over the first half of the century had fundamentally reformulated what it meant to do mathematics. Other mathematicians saw them as ephemeral research fads, divorced from the main historical trends of mathematical development. Tied to broader criticism surrounding the rise of the "Bourbaki program," as well as to insecurities among so-called applied mathematicians, the controversy over the new math curriculum in the late 1950s and early 1960s reveals the surprising instability of the identity of the mathematician. Although midcentury curriculum reform was an international phenomenon, the debate over the curriculum, and over the definition and role of applied mathematics, was especially poignant and volatile within the United States.[11]

Despite the new curriculum's origins as the product of professional mathematicians, SMSG's goals did not include training more mathematicians. Rather, the group wanted to reform the way students learned to think by reforming the way math was taught. The sales pitch was that the new math rejected rote memorization in favor of mathematics taught as structured reasoning. Mathematicians didn't memorize facts or solve calculation problems—they analyzed structures—and teaching children to do the same would promote disciplined intelligence generally. In turn, SMSG's critics claimed that the organization spoke only for a minority of mathematicians and mischaracterized the field as a whole. Notwithstanding broad agreement that students should learn to think mathematically—a view pervasive enough to justify federal funds to pay mathematicians to improve the nation's school textbooks—mathematicians simply could not agree on the nature of their practice or the source of mathematics' powerful applications. The new math controversy was never about whether the math in the books was *correct* but, instead, about the authors' attempt to base mathematical training on a particular view of the discipline. This debate

Many articles in *Mathematics Teacher* attest to some teachers' interest in reform; see, e.g., Howard F. Fehr, "Helping Our Teachers," *Mathematics Teacher*, 1957, *50*:451–452. There is also some evidence that high school teachers resented the fact that federal money appeared only after mathematicians took charge. See Bruce Meserve to Edward Begle, 12 Oct. 1965, Folder "NCTM 1965–1966," Box 13.6/86-28/60, SMSGR.

[10] The Cambridge Conference on School Mathematics was another group closely aligned with professional mathematicians, but its recommendations were never translated into widely used textbooks. See Cambridge Conference on School Mathematics, *Goals for School Mathematics* (Boston: Houghton Mifflin, 1963). For the Illinois group, in particular, see Max Beberman, *An Emerging Program of Secondary School Mathematics* (Cambridge, Mass.: Harvard Univ. Press, 1958); and Thomas Steven Dupre, "The University of Illinois Committee on School Mathematics and the 'New Mathematics' Controversy" (Ph.D. diss., Univ. Illinois, 1986).

[11] For a survey of international programs see Bob Moon, *The "New Maths" Curriculum Controversy* (London: Falmer, 1986). Reforms in other national contexts, where curriculum development and adoption is centralized, experienced less controversy and rancor when compared with the American case.

over the new math was ultimately a battle over the definition of mathematical knowledge itself.

NEW IMAGES OF MATHEMATICS

The identification of mathematics with reasoning in general was certainly not limited to the postwar period. Mathematics has long been assumed to inculcate powers of reckoning and instantiate certain knowledge. Recent historical work has begun to emphasize the ways this link was made. Reviel Netz has argued that ancient Greek deductive geometry, with its diagrams, highly specific language, and strict conventions, was related to an "idealized, written version of oral argument." Netz's *The Shaping of Deduction in Greek Mathematics* concludes that mathematicians were likely "eccentrics" in a "world of doctors, sophists, and rhetoricians"—but eccentrics who established stable practices for making *formal* rhetorical arguments. Greek geometry, the foundation of mathematical reasoning for centuries, was originally a practice impossible to understand in isolation from contemporary intellectual and material tools of persuasive reasoning.[12]

Over the succeeding centuries, whether inside or outside of formal pedagogical institutions, mathematics continued to be firmly tied to intellectual and philosophical practices. Math was a crucial resource for conceptualizing thought. Matthew Jones's work has shown how René Descartes, Blaise Pascal, and Gottfried Leibniz all conceived of mathematics as a way of cultivating the self. Mathematical techniques improved the ability of humans to reason and examine evidence, these men claimed, but also revealed the limitations of human knowledge. Near-contemporaries opposed extensive training in or reliance on mathematics with precisely the opposite justification. Math, Roger Ascham, Robert Boyle, and others suggested, was unsuited for natural philosophy or intellectual training. Likewise, in eighteenth-century France and nineteenth-century Britain, debates about the nature of mathematics were often resolved on the basis of how mathematics was understood to discipline students' minds. One's view of the patient ostensive methods of geometry or the rapid analytical techniques of algebra—with their accompanying pedagogical commitments—depended on one's view of the kind of thinking that ought to be cultivated in students.[13]

In the mid-twentieth-century American context, mathematics and reasoning remained tightly associated. After the great success of the mathematical sciences in World War II, and with mathematics' increasing incorporation into the bureaucratic and political func-

[12] Reviel Netz, *The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History* (Cambridge: Cambridge Univ. Press, 1999), esp. pp. 309–312.

[13] This literature is vast. See, e.g., Matthew L. Jones, *The Good Life in the Scientific Revolution: Descartes, Pascal, Leibniz, and the Cultivation of Virtue* (Chicago: Univ. Chicago Press, 2006); Roger Ascham, *The Scholemaster* (London, 1579), p. 6; Steven Shapin, *A Social History of Truth: Civility and Science in Seventeenth-Century England* (Chicago: Univ. Chicago Press, 1994), esp. pp. 315–352; Peter Dear, *Discipline and Experience: The Mathematical Way in the Scientific Revolution* (Chicago: Univ. Chicago Press, 1995); George Elder Davie, *The Democratic Intellect: Scotland and Her Universities in the Nineteenth Century* (1961), 2nd ed. (Edinburgh: Edinburgh Univ. Press, 1964); Richard Olson, "Scottish Philosophy and Mathematics, 1750–1830," *Journal of the History of Ideas*, 1971, *32*:29–44; Andrew Warwick, *Masters of Theory: Cambridge and the Rise of Mathematical Physics* (Chicago: Univ. Chicago Press, 2003); Joan L. Richards, "Historical Mathematics in the French Eighteenth Century," *Isis*, 2006, *97*:700–713; and Richards, *Mathematical Visions: The Pursuit of Geometry in Victorian England* (Boston: Academic, 1988). On changing images in mathematics more generally see Bottazini and Dahan Dalmedico, eds., *Changing Images in Mathematics* (cit. n. 4); and Amir Alexander, *Duel at Dawn: Heroes, Martyrs, and the Rise of Modern Mathematics* (Cambridge, Mass.: Harvard Univ. Press, 2010).

tioning of the state in the guise of computing, game theory, rational choice theory, and operations research, math remained a powerful model and methodology for the creation of secure knowledge. Illinois Senator Charles Percy spoke for many when he explained in 1958 that "we need the fullest possible development of the capacity to think, to reflect, to weigh and judge, to make choices among alternatives, and to foresee the consequences of these choices. This is the modern mind we need—the mind of the scientist, the key executive, the mathematician." The American "liberal consensus" in the 1950s and early 1960s, however fragile, clearly emphasized the important role and power of state expertise, particularly expertise derived from mathematics and the sciences.[14]

SMSG was representative of just such expertise. Although only a few mathematicians were closely associated with SMSG, they were powerfully located within the mathematical establishment, including A. Adrian Albert (Chicago), Andrew Gleason (Harvard), Albert Meder, Jr. (Rutgers), Edwin Moise (Michigan/Harvard), Albert Tucker (Princeton), Marshall Stone (Chicago), and G. Baley Price (Kansas). SMSG was an organization run by—and intended to produce material representative of—elite research mathematicians.

Research mathematicians, though, were not supposed to care about school mathematics. American academic mathematicians had long *defined* themselves as removed from the problems of education. That was, after all, why professional mathematicians splintered into three organizations in the first decades of the twentieth century. The American Mathematical Society (AMS) was established for professional research, the Mathematical Association of America to serve as a central organization for the teaching of college-level mathematics, and the National Council of Teachers of Mathematics to advocate for high school mathematics instruction.[15] SMSG started as a collaboration of these societies, and membership on its initial steering committee was drawn from all of them. The unusual

[14] U.S. Congress, Senate, Committee on Labor and Public Welfare, *Science and Education for National Defense*, 85th Cong., 2nd sess. (28 Mar. 1958), p. 1373 (Percy). On science and expertise see, among many, S. M. Amadae, *Rationalizing Capitalist Democracy: The Cold War Origins of Rational Choice Liberalism* (Chicago: Univ. Chicago Press, 2003); Jennifer S. Light, *From Warfare to Welfare: Defense Intellectuals and Urban Problems in Cold War America* (Baltimore: Johns Hopkins Univ. Press, 2003); Paul Erickson, Judy L. Klein, Lorraine Daston, Rebecca Lemov, Thomas Sturm, and Michael D. Gordin, *How Reason Almost Lost Its Mind: The Strange Career of Cold War Rationality* (Chicago: Univ. Chicago Press, 2013); Erickson, "Mathematical Models, Rational Choice, and the Search for Cold War Culture," *Isis*, 2010, *101*:386–392; Frank Newman, "The Era of Expertise: The Growth, the Spread, and Ultimately the Decline of the National Commitment to the Concept of the Highly Trained Expert, 1945 to 1970" (Ph.D. diss., Stanford Univ., 1981); Brian Balogh, *Chain Reaction: Expert Debate and Public Participation in American Commercial Nuclear Power, 1945–1975* (New York: Cambridge Univ. Press, 1991); Philip Mirowski, *Machine Dreams: Economics Becomes a Cyborg Science* (Cambridge: Cambridge Univ. Press, 2002); Agatha C. Hughes and Thomas P. Hughes, *Systems, Experts, and Computers: The Systems Approach in Management and Engineering, World War II and After* (Cambridge, Mass.: MIT Press, 2000); Hunter Crowther-Heyck, *Herbert A. Simon: The Bounds of Reason in Modern America* (Baltimore: Johns Hopkins Univ. Press, 2005); Paul N. Edwards, *The Closed World: Computers and the Politics of Discourse in Cold War America* (Cambridge, Mass.: MIT Press, 1996); and David Raymond Jardini, "Out of the Blue Yonder: The RAND Corporation's Diversification into Social Welfare Research, 1946–1968" (Ph.D. diss., Carnegie Mellon Univ., 1996). On "liberal consensus" see Godfrey Hodgson, *America in Our Time* (Garden City, N.Y.: Doubleday, 1976). The degree of actual consensus was a matter of debate, particularly on divisive issues like race; see Gary Gerstle, "Race and the Myth of the Liberal Consensus," *Journal of American History*, 1995, *82*:579–586.

[15] Alan R. Osborne and F. Joe Crosswhite, "Forces and Issues Related to Curriculum and Instruction, 7–12," in *A History of Mathematics Education in the United States and Canada: Thirty-Second Yearbook* (Washington, D.C.: National Council of Teachers of Mathematics, 1970), pp. 155–300, esp. pp. 194–196; Karen Hunger Parshall and David E. Rowe, *The Emergence of the American Mathematical Research Community, 1876–1900: J. J. Sylvester, Felix Klein, and E. H. Moore* (Providence, R.I.: American Mathematical Society, 1994), pp. 401–419; and David Lindsay Roberts, *American Mathematicians as Educators, 1893–1923: Historical Roots of the "Math Wars"* (Chestnut Hill, Mass.: Docent, 2012).

involvement of the AMS in school mathematics did not go unnoticed at the time, and some mathematicians raised the concern that research mathematicians shouldn't bother with precollegiate curricular issues. John Green, the secretary of the AMS and a mathematician at UCLA, for example, wrote AMS president (and Harvard professor) Richard Brauer that curricular development is plainly "not Society business." Brauer's response was telling, suggesting not only that other efforts had failed to produce satisfactory curricula but also that the AMS should never have abdicated responsibility for pedagogy in the first place and that the society should now "accept the responsibility for clearing up the present mess."[16] SMSG's very existence testifies to the fact that mathematicians in this period believed they should play a substantial role in educational reform, ensuring that the discipline was portrayed properly.

SMSG's portrayal of the nature of mathematics was at least partially based on work that had already been done by the College Board's Commission on Mathematics, and Begle explicitly encouraged SMSG writers to take the commission's report as a "reasonable starting point." Headed by the Princeton mathematician Albert Tucker, the commission began work in 1955; its report, published four years later, described the nature of mathematical knowledge using an analogy:

> As a city grows, it becomes increasingly difficult to find adequate transportation from the center of the city to the outlying areas and the suburbs. The center is still the core of the city but the streets are too narrow and too congested for the newer sections to be reached as quickly as the needs of the residents require. For a time, systems of traffic lights and one-way streets suffice; but ultimately these patchwork methods, too, are found to be inadequate. Then there is constructed a limited-access freeway or expressway from the heart of the city to outlying points, bringing the newer regions effectively closer to the core.

The same was true of the changes in mathematical knowledge. "New and more efficient routes" in the foundations of the subject have been found that enable students to reach "modern mathematics" without "laboriously traversing all of the older content."[17] The claim was about both mathematics and pedagogy. Mathematics, like a city, changed not only by addition, but also structurally; and if they were taught the newer and more fundamental structures, students might learn to see and navigate the whole more easily.

The analogy implied a certain degree of destructiveness. Midcentury cities did not keep all the old along with the new but bulldozed some less useful structures in favor of new ones. Mathematics had to do this as well—subjects might be eliminated from areas of active research or teaching, whether or not they had been proven "wrong." Certain topics might simply no longer be worth pursuing—or they might have been replaced by more powerful, direct, or efficient routes.

The report's authors had lifted this metaphor—without acknowledgment—from an article by "Nicolas Bourbaki." Writing in the late 1940s (with an English translation

[16] John W. Green to Richard Brauer, 7 Mar. 1958, and Brauer to Green, 10 Mar. 1958, Folder 54, Box 45, American Mathematical Society Archives, Brown University, Providence, Rhode Island. Thanks to Alma Steingart for drawing my attention to this correspondence.

[17] College Entrance Examination Board, *Report of the Commission on Mathematics: Program for College Preparatory Mathematics* (New York: College Entrance Education Board, 1959), pp. 5–6. For the College Board's influence on SMSG see Folder "Speeches—E. G. Begle," Box 4.1/86-28/48, SMSGR; Guilford Spencer II of Williams College, in SMSG, *Report of an Orientation Conference for Geometry with Coordinates, Chicago, Illinois, September 23, 1961* (Stanford, Calif.: SMSG, 1962), p. 29; and E. E. Moise's report in Folder "History and Philosophy of SMSG Writing Teams," Box 13.7/86-28/81, SMSGR, p. 25.

appearing in 1950 in the *American Mathematical Monthly*), Bourbaki argued that it was time to reexamine the "internal life of mathematics" by noting that it was like "a big city, whose outlying districts and suburbs encroach incessantly, and in a somewhat chaotic manner, on the surrounding country, while the center is rebuilt from time to time, each time in accordance with a more clearly conceived plan and a more majestic order, tearing down the old sections with their labyrinths of alleys, and projecting towards the periphery new avenues, more direct, broader and more commodious." Like the College Board commission's report, Bourbaki claimed that new routes should replace old labyrinths, enabling one to move from the center to the newest regions most efficiently. Postwar mathematics and contemporaneous urban renewal shared an impetus: new structures justified some "tearing down" in order to form a "more majestic order."[18]

Borrowing the language of Bourbaki meant taking sides in the mathematical world. "Nicolas Bourbaki" was the pseudonym for a well-known and controversial group of mathematicians who were interested in completely reformulating the image of mathematics. With sensibilities forged by the intellectual crucible of L'École Normale Supérieure in the 1920s, mathematicians including Henri Cartan, Claude Chevalley, Jean Delsarte, Jean Dieudonné, René de Possel, and André Weil formed Bourbaki in the following decade. If the group's initial goal of writing an analysis textbook in the "modern" style was rather modest, the project grew in ambition over the course of the 1930s, resulting in a series of volumes under the grandiose title *Éléments de mathématique*. Replacing the language of geometry and spatial intuition with one reliant on formal algebraic structures, the group stood for a new idea of what it meant to practice mathematics, even if its direct influence on mathematicians has often been overstated.[19]

The Bourbaki collaborators' choice of a textbook for their central project indicates the importance of pedagogical concerns. Not content with just changing the direction of research, Bourbaki's members pointed to the ways recent research had fundamentally changed mathematicians' understanding of the nature of mathematical knowledge and thereby revolutionized the way math should be taught. The group held talks for high school teachers in Paris through the mid 1950s, arguing for a move away from traditional subject material in mathematics. Bourbaki's effect on contemporaries outside of France was occasionally diffuse and indirect but still important. The Swiss psychologist Jean Piaget formulated his theory of the stages of intellectual development by citing a Bourbaki-inspired notion of "modern" math: "there exists, as a function of the development of intelligence as a whole, a spontaneous and gradual construction of elementary logico-mathematical structures and . . . these 'natural' ('natural' the way that one speaks of the 'natural' numbers) structures are much closer to those being used in 'modern'

[18] Nicholas Bourbaki, "The Architecture of Mathematics," *American Mathematical Monthly*, 1950, *57*:221–232, on p. 230; the article was originally published as Nicolas Bourbaki, "L'architecture des mathématiques," in *Les grands courants de la pensée mathématique*, ed. F. Le Lionnais ([Marseilles:] Cahiers du Sud, 1948), pp. 35–47, on p. 45. For the relevance of the structural metaphor in a different setting see Peter Galison, "Structure of Crystal, Bucket of Dust," in *Circles Disturbed: The Interplay of Mathematics and Narrative*, ed. Apostolos Doxiadis and Barry Mazur (Princeton, N.J.: Princeton Univ. Press, 2012), pp. 52–78.

[19] On the overstatement see Leo Corry, *Modern Algebra and the Rise of Mathematical Structures* (Basel: Birkhäuser, 1996), esp. Ch. 7. On Bourbaki generally see Maurice Mashaal, *Bourbaki: A Secret Society of Mathematicians*, trans. Anna Pierrehumbert (Providence, R.I.: American Mathematical Society, 2006); David Aubin, "The Withering Immortality of Nicolas Bourbaki: A Cultural Connector at the Confluence of Mathematics, Structuralism, and the Oulipo in France," *Science in Context*, 1997, *10*:297–342; Henri Cartan, "Nicholas Bourbaki and Contemporary Mathematics" (1958), *Mathematical Intelligencer*, 1980, *2*:175–187; Joong Fang, *Bourbaki* (Hauppauge, N.Y.: Paideia, 1970); and Liliane Beaulieu, "Bourbaki's Art of Memory," *Osiris*, 2002, N.S., *14*:219–251.

mathematics than to those being used in traditional mathematics." Piaget's research, particularly as reformulated through the Harvard psychologist Jerome Bruner, was in turn a crucial influence on groups like SMSG. Bruner infamously claimed that "any subject can be taught effectively in some intellectually honest form to any child at any stage of development"; more relevant to SMSG was that he based his pedagogical views in part on the idea that "the teaching and learning of structure," instead of "simply the mastery of facts and techniques, was at the center of the classic problem of transfer." Structure, in the Bruner and Piaget sense, was the key to learning mathematics such that its mental habits could be transferred to other domains, a crucial goal of SMSG's curriculum.[20] For SMSG and Bourbaki, the structures of modern math should determine the emphasis of both research and pedagogy.

Bourbaki's emphasis on the elimination of the "labyrinths" of outdated methods also influenced curriculum reformers. In a 1959 speech, one of Bourbaki's most outspoken members, Jean Dieudonné, proclaimed that "Euclid must go!" For Dieudonné, Euclidean geometry was "dead weight," with few interesting questions and little relevance. The useful bits might be profitably taught in a few hours, and the rest of geometry has "as much relevance to what mathematicians (pure and applied) are doing today as magic squares or chess problems."[21] Mathematicians could find better ways to teach the rigorous—logical, analytical—methods of mathematics. Dieudonné speculated that only faith and tradition had kept Euclid alive in the curriculum. While he focused his attack on geometry in particular, it stood for all outmoded subjects: as the College Board's commission recommended that same year, it was high time to lay down new and more efficient routes from the core to the periphery of mathematics.

Dieudonné's speech provoked sharp controversy. He was speaking at the Royaumont Conference, an international gathering concerning reform of the mathematics curriculum. Even among reformers, few were willing to countenance the near-complete elimination of Euclidean geometry from the school curriculum. Nevertheless, Dieudonné's provocative claims were influential among the Americans who were in attendance: Albert Tucker of Princeton and the College Board, Howard Fehr of Teachers College, and three members of SMSG, Edward Begle, Robert Rourke, and Marshall Stone. Stone had, in fact, delivered the opening address of the conference, encouraging precisely the sort of "modernization" of the curriculum that Dieudonné's views represented. One of SMSG's geometry writers later claimed that some of the group's work was a "partial answer" to Dieudonné's "Euclid must go" speech. Looking back a decade later, the French mathematician René Thom suggested that it would be only natural for mathematicians "steeped in the ideas of Bourbaki" to write textbooks like SMSG's.[22]

[20] See, e.g., H. Cartan *et al.*, *Structures algébriques et structures topologiques* (Geneva: L'Enseignement Mathématique, 1958) (a collection of essays presented at conferences for teachers of mathematics at the Henri-Poincaré Institute at the Sorbonne, in Paris, from Feb. 1956 to June 1957); Jean Piaget, "Comments on Mathematical Education," in *Developments in Mathematical Education: Proceedings of the Second International Conference on Mathematical Education*, ed. A. G. Howson (Cambridge: Cambridge Univ. Press, 1973), pp. 79–87, on p. 79; and Jerome Bruner, *The Process of Education* (1960; New York: Vintage, 1963), pp. 12, 33.

[21] Jean Dieudonné's speech was published in Organization for European Economic Co-Operation, *New Thinking in School Mathematics* ([Paris:] OEEC, 1961); for the quotations see pp. 35–36.

[22] SMSG, *Report of an Orientation Conference for Geometry with Coordinates* (cit. n. 17), esp. pp. 12–15; and René Thom, "'Modern' Mathematics: An Educational and Philosophic Error," *American Scientist*, 1971, *59*:695–699, on p. 695.

SMSG AND MATHEMATICS AS STRUCTURE

While Begle's appointment had been largely happenstance—he was looking for a way to move into administration full time just as the AMS needed someone to head SMSG—it resulted in an intellectual trajectory that rooted SMSG's conception of mathematical knowledge in Bourbaki-inspired soil. Begle's mathematical background was in topology, as was that of Harvard's Andrew Gleason, an SMSG-allied reformer who later recalled that he had likely been influenced by Bourbaki's emphasis on abstraction and structure. Another key SMSG supporter, A. Adrian Albert, was the author of a book called *Structure of Algebras*. Perhaps recognizing the hyperbole within Bourbaki's rhetoric, Begle never explicitly proselytized for that group's vision, but he was nonetheless a believer in their model of mathematics and echoed them when he defined mathematics as a "set of *interrelated, abstract, symbolic systems*."[23] Although writing teams occasionally rejected Begle's advice and SMSG had semi-independent advisory boards, Begle and his small group of close allies retained a great deal of power and ultimately controlled the direction of SMSG.

Of SMSG's central mathematician-writers, the most outspoken proponent of Bourbaki was the University of Chicago's Marshall Stone. Stone is widely credited with transforming Chicago's mathematics department in the late 1940s, with the stated goal, as he later characterized it, of "elaborating a modernized curriculum."[24] One of his key hires in this respect was André Weil, a French mathematician deeply involved with Bourbaki. Stone's own research into Boolean algebras focused on structures and classes rather than on the particular characteristics of elements themselves, a classic Bourbaki approach. His work in the 1930s on Boolean algebras proved to be a "central milestone" for much of twentieth-century mathematics, in the words of the historian Leo Corry. Over the subsequent two or three decades, many mathematicians took for granted that the work of algebra was to reveal underlying structures. Stone's research suggested the productivity of bringing the structure of mathematics to the fore, both in solving problems and in opening new areas of research, and he worked to bring similarly minded mathematicians to positions of prominence in the United States.[25]

In 1961, Stone published an article in *American Mathematical Monthly* explaining his view of the relationship between scientific and mathematical knowledge and its implication for curricular reform efforts. Stone argued that "modern mathematics" was truly "abstract" mathematics, detached from physical applications. Mathematicians, Stone suggested, needed to "rethink" their "entire conception of education," since a "quiet revolution" had taken place separating mathematics from the physical world. It was now possible to "provide a nearly final answer to the question 'What is Mathematics?'" because "a modern mathematician would prefer the characterization of his subject as the study of general abstract systems, each one of which is an edifice built of specific abstract elements

[23] Interview with Andrew Gleason, transcript, Box 4RM17, R. L. Moore Legacy Collection, 1890–1900, 1920–2009, Archives of American Mathematics, pp. 51–52; A. Adrian Albert, *Structure of Algebras* (New York: American Mathematical Society, 1939); and E. G. Begle, *Critical Variables in Mathematics Education: Findings from a Survey of the Empirical Literature* (Washington, D.C.: Mathematical Association of America and National Council of Teachers of Mathematics, 1979), p. 1.

[24] Marshall H. Stone, "Reminiscences of Mathematics at Chicago," in *A Century of Mathematics in America*, ed. Peter Duren, 3 vols. (1976; Providence, R.I.: American Mathematical Society, 1989), Vol. 2, pp. 183–190, on p. 185. See also Saunders Mac Lane, "Mathematics at the University of Chicago: A Brief History," *ibid.*, pp. 127–154; and Felix E. Browder, "The Stone Age of Mathematics on the Midway," *ibid.*, pp. 191–193.

[25] Corry, *Modern Algebra and the Rise of Mathematical Structures* (cit. n. 19), pp. 289, 337, 355–357.

and structured by the presence of arbitrary but unambiguously specified relations among them." Stone proceeded to survey the various fields of mathematics and show how a "fundamental unity" held. While not neglecting the "manipulative aspects" of the subject or the importance of those "who engage in making applications of mathematics," curriculum designers should push forward "abstraction and the discernment of patterns" as the key element of mathematical practice.[26]

These claims were ultimately inscribed into the textbooks SMSG produced. One could draw examples from any number of them, but the most influential textbooks were those for junior high, geometry, and algebra courses, written between 1958 and 1961. SMSG's books represented a rejection of mathematics as either a fixed set of skills for practical application or a list of facts to be memorized. In this, the group's writers were following the lead of Stone and many others in de-emphasizing math as primarily skills and facts. Detlev Bronk, president of the National Academy of Sciences in the 1950s and then chair of the National Science Board of the NSF, complained around the time of SMSG's founding that in most mathematics books "there is too much emphasis upon facts, and too little emphasis on training in the ability to think." The NSF later elaborated what methodological role a new mathematics curriculum might play when it explained the origins of SMSG in its annual report. "Mathematics today," the report suggested, "is an entirely different discipline from what it was even a generation ago. Its applications have been so extended that scientists in many new fields use it as physicists and engineers used it early in the twentieth century." As a result, SMSG would be formed to make mathematics "a way of thinking rather than a system of artificial devices to solve problems." Math was not widely useful because of some particular set of facts or processes but because it was itself a way of thought. Stone himself said as much in a 1957 speech: "science is reasoning; reasoning is mathematics; and therefore, science IS mathematics."[27] Modern math is a model of rigorous thought, so one just needs to introduce students to the way in which mathematicians think.

SMSG's mathematicians complained that most previous textbooks implied that mathematicians calculated. According to SMSG, whatever else they might have done, mathematicians certainly *did not* calculate. The junior high school series begins with a story of a mathematician telling his neighbor on a plane that he did not, in fact, do calculations with figures all day long—that was best done by a machine. Rather, "my job," the mathematician said, "is mainly logical reasoning." That meant "deductive reasoning," or reasoning from "if–then" statements.[28] The first problems students faced involved logic puzzles that required neither counting nor measuring but, instead, careful reasoning about particular sets of information. Such problems attempted to convince students that math was more than an ever-expanding set of computational skills.

Moreover, mathematics was not a "completed" subject, analogous to Latin grammar, but, rather, an ongoing and expanding field of active research. "In the past 50 years," the SMSG junior high text proclaimed, "more mathematics has been discovered than in all the

[26] Marshall Stone, "The Revolution in Mathematics," *Amer. Math. Mon.*, 1961, *68*:715–734, on pp. 715–717, 730, 733.

[27] U.S. Congress, Senate, Committee on Labor and Public Welfare, *Science and Education for National Defense*, 85th Cong., 2nd sess. (21 Jan. 1958), pp. 6–7 (Bronk); National Science Foundation, *Eighth Annual Report for Year Ended June 30, 1958* (Washington, D.C.: Government Printing Office, 1958), pp. 64–66; and Marshall Stone, quoted in Dahan Dalmedico, "Image Conflict in Mathematics after 1945" (cit. n. 4), p. 232.

[28] SMSG, *Mathematics for Junior High School: Student's Text*, rev. ed., 2 vols. (New Haven, Conn.: Yale Univ. Press, 1961), Vol. 1, p. 1.

preceding thousands of years of man's existence." Fittingly, the junior high curriculum ends with a chapter on unsolved problems, an attempt to dispel the idea that mathematics is a "dead and completed subject that was embalmed between the covers of a textbook sometime after Sir Isaac Newton."[29]

Instead of emphasizing calculation facts, SMSG's writers positioned mathematics as a mode of thought—a very specific mode, indeed, derived from the underlying nature of mathematics itself. That is, SMSG argued that mathematics was built systematically and that, by learning its structures, students would come to reason mathematically and therefore reliably. They explained this to teachers by introducing two "planes" of reasoning, one of "raw data" and one of mathematical concepts. To come to reliable conclusions, one had carefully to choose postulates and rules for the conceptual plane, translate the messy world of raw data into this plane, reason logically to conclusions, and then translate conclusions back into facts about the world. The analysis of mathematical structures provided training in just this sort of reasoning. SMSG's approach diverged from the traditional view of, say, Euclidean geometry as a model to be emulated stylistically or of algebra as a set of formal symbolic techniques. No longer were such rigid methods of demonstration crucial. Whereas previous curriculum designers had emphasized the importance of specific techniques or methods—perhaps best represented by the well-known (if little followed) 1923 Mathematical Association of America report promoting "functional thinking" through an understanding of mathematical functions—SMSG began to assert the importance of learning about the underlying nature of mathematics itself. SMSG also differed from other near-contemporary efforts to define the relationship between mathematics and reasoning—such as that of George Pólya, who claimed that mathematics provided students with a general set of problem-solving heuristics but did not base his arguments on the underlying structure of the field.[30] SMSG's authors believed that math could provide the right kind of disciplined intelligence for the twentieth century only if school textbooks portrayed the subject as mathematicians understood it. Mastering the underlying structure of mathematics would provide students with a powerful approach for moving from intuitions and hunches to rigorous, reliable conclusions in all sorts of domains.

SMSG's version of mathematics as systematic reasoning was embedded within the junior and senior high school textbooks. The authors introduced modular arithmetic as a "new kind" of calculation—namely, an alternative mathematical "system," or "a set of elements together with one or more binary operations defined on the set."[31] (See Figure 1.) Modular arithmetic, like the common arithmetic students had already learned, exemplified a mathematical system with its own elements, operations, and properties.

After explaining changes of base in order further to emphasize the relationship between

[29] *Ibid.*, p. 8; and SMSG, *Mathematics for Junior High School: Student's Text*, Vol. 2, p. 545.

[30] SMSG discussed the two "planes" of reasoning in SMSG, *Mathematics for High School: Geometry: Teacher's Commentary*, rev. ed. (New Haven, Conn.: Yale Univ. Press, 1961), pp. 518–520. On the early twentieth-century reforms see Roberts, *American Mathematicians as Educators, 1893–1923* (cit. n. 15). Although Begle's wife later said that SMSG's leader admired Pólya greatly, Pólya did not seem to appreciate SMSG's textbooks: interview with Elsie Begle, transcript, Box 4RM15, R. L. Moore Legacy Collection, 1890–1900, 1920–2009, p. 15. For Pólya's views see George Pólya, *How to Solve It: A New Aspect of Mathematical Method* (Princeton, N.J.: Princeton Univ. Press, 1945); and Pólya, *Induction and Analogy in Mathematics*, 2 vols. (Princeton, N.J.: Princeton Univ. Press, 1954).

[31] SMSG, *Mathematics for Junior High School: Student's Text* (cit. n. 28), Vol. 1, p. 559. "Binary" in that there are two inputs: i.e., "2x3=6" represents a binary operation because ordinary multiplication uses two elements to produce a third; other operations, like "take the derivative of *f(x)*," are not binary.

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3. Here is a table for addition (mod 5).

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Copy and complete the following table.

(Mod 5)

b	a	additive inverse of a	b - a	b + additive inverse of a
0	1	4	0 - 1 ≡ 4	0 + 4 ≡ 4
2	1		2 - 1 ≡	2 + 4 ≡
4	1			
1	2			
2	2			
3	2			
2	4			
3	4			
4	4			

Figure 1. *Through modular arithmetic, SMSG's textbooks asked students to explore the structures underlying mathematical computation, even when the calculations differed from familiar examples. From SMSG,* Mathematics for Junior High School: Student's Text, *rev. ed., 2 vols. (New Haven, Conn.: Yale Univ. Press, 1961), Vol. 1, p. 557.*

the usual arithmetic representations and underlying structural elements, SMSG suggested that mathematicians might not even work with numbers at all. The possible spatial manipulations of a playing card, for instance, constituted a mathematical system because one can think of the various orientations of the card as the elements and the procedures of flipping and rotating as operations transforming one element (orientation) into another. (See Figure 2.) As with the previous examples, properties and concepts from elementary arithmetic like "inverse" and "commutativity" were discussed. Elements—whether numbers or not—and operations formed "systems," the properties of which could be studied and elucidated. Relatedly, sets and set notation were deployed starting in first grade to emphasize the systems and logical operations that grounded arithmetic, algebra, and geometry. Mathematical reasoning involved defining and exploring systems, not learning the facts of one particular method of calculation.

The effort to present math as a general exploration of systems implied that differences between subjects like algebra and geometry were no longer as important as the way they both instantiated mathematical reasoning. Traditionally, the analytical methods of algebra allowed students to manipulate symbols mechanically or experimentally to arrive at new relationships between quantities, while geometry emphasized using primitive postulates to construct propositions about spatial figures systematically. Although geometric and algebraic techniques have been intertwined in mathematical research at least since the seventeenth century, the distinction remained pedagogically important. By the time SMSG released algebra and geometry textbooks in the early 1960s, however, its writers wanted

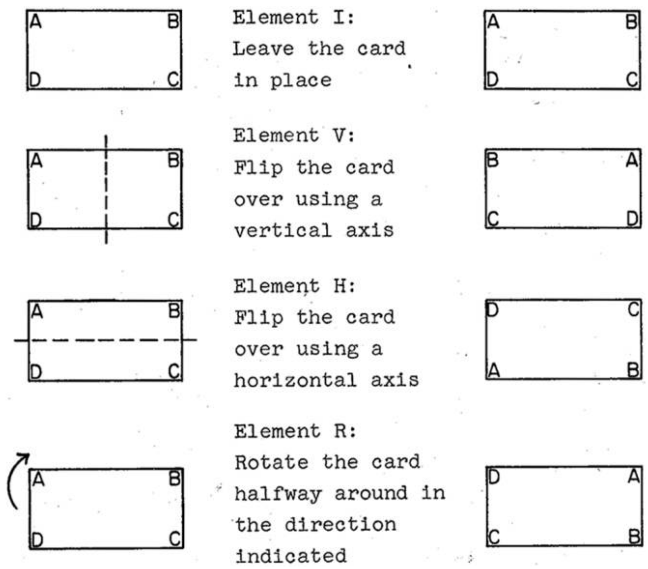

Figure 2. *For SMSG, mathematics was not fundamentally about numerical computation, and the group's textbooks showed how mathematical operations might involve, for example, the spatial arrangements of a card. From SMSG,* Mathematics for Junior High School: Student's Text, *rev. ed., 2 vols. (New Haven, Conn.: Yale Univ. Press, 1961), Vol. 1, p. 566.*

to emphasize the *unity* of mathematical reasoning, not its variety. The geometry text, the teachers' commentary announced, introduced algebraic concepts and techniques early on, because "a great deal of the traditional material of geometry *was really algebraic all along*." The geometric postulates given to students freely mixed concepts taken from both subjects—for example, in the correspondence between a geometric line and the real numbers.[32] SMSG's presentation implied a fundamental shift away from distinguishing geometry from algebra. Geometry was just one more form of mathematical reasoning, not a privileged method of revealing spatial truths.

Such pedagogical choices were predicated on broader disciplinary developments, especially those concerning Euclidean geometry. While Euclid's theorems had long been synonymous with incontrovertible spatial truths, the proliferation of non-Euclidean geometries in the nineteenth century challenged such assumptions. By the early twentieth century, the American mathematician Oswald Veblen had to specify—because it could no longer be assumed—that his geometric axioms "codified in a definite way our spatial judgments." Slightly later, David Hilbert openly questioned geometry's connection with spatial experience by noting that while "Two points define a line" is a geometric statement, our terms "points," "lines," and "planes" could just as easily be replaced by "tables," "chairs," and "beer mugs." By the 1920s and 1930s, the Harvard mathematician George D. Birkhoff coauthored a school textbook formed from his own axioms, which emphasized that geometry was about codifying general reasoning, not about a style of argument inherited from ancient Greece. Rather, the "prime concern" was simply "to

[32] SMSG, *Mathematics for High School: Geometry: Teacher's Commentary* (cit. n. 30), p. 12; and SMSG, *Mathematics for High School: Geometry: Student's Text* (New Haven, Conn.: Yale Univ. Press, 1961), Ch. 2 and p. c of the appendixes.

make the students articulate about the sort of thing that hitherto [they have] been doing quite unconsciously."[33] The achievements of Veblen, Hilbert, and Birkhoff laid the intellectual groundwork for SMSG's *Geometry* to draw freely from algebra and other disciplines. It provided simply one more example of—and training in—structured mathematical knowledge, not a list of spatial facts or an ossified form of reasoning.

Likewise, the authors of the algebra text emphasized its similarity to geometry by explaining that "all" mathematics proceeded by separating out assumed properties as axioms or postulates and constructing theorems and corollaries. SMSG's textbooks attempted to avoid the case in which algebra simply "degenerates into just so much formalism." Its writers did this in two ways. First, they suggested that the techniques of algebra were useful only insofar as they revealed the structure of numbers. Rather than thinking of the techniques as meaningless symbolic games, algebraic simplification allowed students to understand the underlying relations between numbers and, therefore, the structure of the number system. "The problem [of factoring]," the text tells students, "is to write a given polynomial, which we consider to be of a certain type, as an indicated product of polynomials of the *same* type." The authors emphasized the importance of algebraic types—for example, that "for each value of its variables a rational expression is a real number."[34] Second, general conclusions about numbers required mastery of algebra. Facts about expressions involving properties of rational and real numbers cannot be learned only by calculation—there are simply too many numbers. They require a form of reasoning in general, just as geometric propositions allowed one to move from intuition about one right triangle to a conclusion about all of them.

Proofs consequently entered the introductory algebra textbook to show students how algebra allowed them to know something about numbers in general. After having learned elementary arithmetic, for example, students were supposed to know that "two numbers whose sum is 0 are related in a very special way"—namely, that if one adds a particular number to four and gets zero, then the number is an additive inverse of four. Then, students are asked whether there can be more than one additive inverse for a given number. "All our experience with numbers tells us 'No, there is no other such number.' But how can we be absolutely sure?" The authors suggest that this can be settled algebraically using the underlying properties of the system:

> Suppose z is any additive inverse of x, that is, any number such that $x + z = 0$. . . . We use the addition property of equality to write
>
> $(-x) + (x + z) = (-x) + 0$
>
> We have then that

[33] Richards, *Mathematical Visions* (cit. n. 13); Jeremy Gray, *János Bolyai, Non-Euclidean Geometry, and the Nature of Space* (Cambridge, Mass.: MIT Press, 2004); and Oswald Veblen, "A System of Axioms for Geometry," *Transactions of the American Mathematical Society*, 1904, *5*:343–384, on p. 343. The Hilbert quotation appears, among many other places, in Stewart Shapiro, *Philosophy of Mathematics: Structure and Ontology* (New York: Oxford Univ. Press, 1997), p. 157. Drawing on material developed in 1898–1899, Hilbert published his own volume in 1899, attempting to lay a new foundation for geometry: David Hilbert, *Foundations of Geometry*, trans. Leo Unger (La Salle, Ill.: Open Court, 1971). For Birkhoff's work see George D. Birkhoff, "A Set of Postulates for Plane Geometry, Based on Scale and Protractor," *Annals of Mathematics*, 1932, 2nd Ser., *33*:329–345; and Birkhoff and Ralph Beatley, *Geometry*, preliminary ed. (Boston: Spaulding-Moss, 1933), esp. p. i.

[34] SMSG, *Mathematics for High School: First Course in Algebra: Teacher's Commentary*, rev. ed. (New Haven, Conn.: Yale Univ. Press, 1961), esp. pp. x, 202–203, 341–343; and SMSG, *Mathematics for High School: First Course in Algebra: Student's Text*, rev. ed. (New Haven, Conn.: Yale Univ. Press, 1961), esp. pp. 313, 347, 352.

$((-x) + x) + z = (-x)$
. . .
Then
$0 + z = -x$
And finally
$z = -x$

Algebra enables students to be absolutely sure that every number has only one additive inverse because the symbols can be manipulated algebraically to conclude that given some x, the only additive inverse is $-x$, which is the opposite of x.[35]

Although such proofs are rare in SMSG's *Algebra*, they are at the core of what SMSG wanted to present as the goal of algebraic knowledge: the ability to use symbols to know things *in general* about numbers, in exact analogy to the way geometric proofs would enable students to know *in general* about triangles or circles. The important parts of mathematics—arithmetic, algebra, or geometry—involved learning the method of moving from postulates to conclusions and the underlying structure of mathematical objects that enabled one to do so.

SMSG's math textbooks were not novel in the sense of including new topics. The table of contents was essentially unchanged—there was no inclusion, for example, of projective geometry or abstract algebra. The objects themselves did not much matter, so the traditional topics could serve just as well as newer ones. Rather, the novelty of the books was to recast the standard topics of school mathematics as opportunities for students to learn about the importance of mathematical structures. SMSG wanted teachers to be clear in emphasizing that the goal of the books was to enable students to "achieve some appreciation of the nature of mathematical systems."[36] The books exemplified Begle's—and Bourbaki's—conception of mathematics as a "set of *interrelated, abstract, symbolic systems*."

SMSG'S CRITICS

SMSG's ideas about the nature of mathematics came under scrutiny from mathematicians almost from the start. As early as the fall of 1961, just as SMSG released its first set of revised textbooks, Harvard's George Carrier described a "bitter controversy" that had arisen within the mathematics community over secondary school education.[37] What was taught in the high schools suddenly mattered a great deal to mathematicians. Partly this was a result of SMSG's prominence and authority. Unlike any other math curriculum reform project, SMSG's texts had the imprimatur of the AMS and so took on the appearance of being definitive. This appearance was particularly problematic because SMSG had posited the "actual" underlying nature of mathematics as fundamental to its presentation. Its critics challenged the curriculum on both ontological and pedagogical levels: that mathematics was actually just so and that students ought to learn math in that way.

Carrier's characterization was made as part of a much broader expression of concern

[35] SMSG, *Mathematics for High School: First Course in Algebra: Student's Text*, pp. 135–137.

[36] SMSG, *Mathematics for Junior High School: Teacher's Commentary*, rev. ed., 2 vols. (New Haven, Conn.: Yale Univ. Press, 1961), Vol. 1, p. 381.

[37] George F. Carrier, Richard Courant, Paul Rosenbloom, C. N. Yang, and H. J. Greenberg, "Applied Mathematics: What Is Needed in Research and Education: A Symposium," *SIAM Review*, 1962, *4*:297–320, on p. 300.

over the new curriculum on the part of applied mathematicians. The Society for Industrial and Applied Mathematics convened a discussion in November 1961 on precisely the issue of how mathematics should be taught. The panel included Carrier himself; H. J. Greenberg, of IBM's Watson Research Center; Richard Courant, the head of the New York University (NYU) applied mathematics program; Paul Rosenbloom, a math professor at the University of Minnesota; and C. N. Yang, a Nobel Prize winner for his work on particle and statistical physics. Courant began by lamenting Stone's article in the *American Mathematical Monthly*. For Courant, Stone's vision of mathematics as divorceable from physical reality was a "danger signal." The "fashion" of abstraction was not simply "half" true; it mischaracterized the actual practice of mathematics. It might be easier just to give up the dream of math as a description of "substantive reality," but Courant warned that "the life blood of our science rises through its roots," and "these roots reach down in endless ramification deep into what might be called reality." Therefore math "must not be allowed to split and to diverge towards a 'pure' and an 'applied' variety." Much of the responsibility should be shouldered by teachers of math, who need to avoid "uninspiring abstraction," "isolation of mathematics," and "catechetic dogmatism"; instead they should emphasize a "close interconnection between mathematics, mechanics, physics, and other sciences."[38]

While Greenberg, Yang, and Carrier were broadly supportive of Courant's views, Rosenbloom dissented by suggesting that mathematics' applicative power came from abstraction and structure, not from attention to pre-existing external problems. An SMSG author himself, Rosenbloom quoted the physicist Paul Dirac's complaint that physical reality can be a "straitjacket" for a mathematician's imagination. A mathematician, Rosenbloom suggested, "finds it much more fruitful to create a mathematical model and then look around for a physical interpretation." Rosenbloom explained how he had initially organized the junior high school SMSG books around the physical applications of the material, but then asked Francis Friedman—a theoretical physicist who was a participant in the NSF's physics curriculum project—to examine them. Friedman told Rosenbloom that the "most practical thing in this book is the chapter on the finite mathematical systems because this shows students how to create new models." Rosenbloom's comments echoed the way SMSG itself explained mathematics' usefulness: "One of the most important activities of modern mathematicians," SMSG's writers told teachers, "is the search for common attributes or properties often found in apparently diverse situations or systems. . . . Frequently, the systems developed out of the intellectual curiosity of mathematicians and their search for patterns in diverse abstract situations have been exactly the tools needed and seized upon by scientists in their attack on the problems of the physical world."[39] Acting out of curiosity, in the abstract and intrinsic "interest" of uncovering properties and structure, mathematicians have found tools that happened to be handy for the study of the physical world. The utility of mathematics, according to Rosenbloom and SMSG, emerged through the way students learned to think structurally and systematically.

The debate was not about the usefulness of mathematics. Wartime mobilizations had demonstrated quite spectacularly the ways seemingly esoteric mathematical sciences could be broadly applied; but because few disciplinary boundaries were respected during

[38] *Ibid.*, pp. 298–299. Courant had espoused similar views many years earlier in the opening pages of his textbook: Richard Courant and Herbert Robbins, *What Is Mathematics?* (New York: Oxford Univ. Press, 1941).

[39] Carrier *et al.*, "Applied Mathematics," pp. 303–304 (the original text misspells "Friedman" as "Freedman"); and SMSG, *Mathematics for Junior High School: Teacher's Commentary* (cit. n. 36), Vol. 1, pp. 381–382.

the war, subsequent institutional arrangements for the burgeoning field of applied mathematicians were largely ad hoc. (In countries where the war mobilization of scientists and mathematicians had been less aggressive, the discipline was even less developed.) More than a decade after the end of the war, MIT's H. P. Greenspan was still addressing the Mathematical Association of America to explain what "Applied Mathematics as a Science" actually meant. Noting the recent initiatives of institutions to create separate programs of applied mathematics, Greenspan attempted to define the field and dispel some of the rumors about it, particularly that applied mathematicians performed only a "technical service." The chair of the 1961 conference on education, IBM's H. J. Greenberg, had similarly opened the discussion by lamenting that applied mathematics was quickly becoming "something of a stepchild" or even an "out-of-step child," "looked upon with equal disinterest by mathematicians, physicists, and engineers." SMSG's approach apparently caused great concern for applied mathematicians trying to establish the relevance of their new field. The Society for Industrial and Applied Mathematics itself had been founded only in 1951, formal "applied mathematics" departments were relatively new in the 1960s, and the conception of an administratively and intellectually distinct field of "applied mathematics" was still precarious. Applied mathematicians were clearly on the defensive in this period, despite the success of wartime mathematics, as the AMS rejected proposals to form special divisions for applied mathematics and Fields Medal committees and international conferences increasingly eschewed "applied" work. SMSG's curriculum provided a venue for those, like Greenspan, who desired to correct the "overemphasis" on "pure mathematics" and reassert the importance of applied mathematics within the school curriculum.[40] Disagreements about SMSG's textbooks pointed directly to divisions within mathematics itself.

SMSG's critics did not focus solely on the group's treatment of mathematics' applications. They also complained that SMSG had misgauged the relationship between mathematical practice and pedagogy. Foremost on this point was one of Courant's colleagues at NYU, Morris Kline. Kline was deeply suspicious about SMSG's textbooks, and he published one of the first critiques of the new math in an October 1961 article. Under the heading "Unqualified Indictment," Kline concluded that "the direction of reform has been almost wholly misguided." Accusing the reformers of desiring change for change's sake, rather than basing their work on evidence of the failures of past instruction, Kline rejected the "old" version of math as a stultifying set of facts and rules but did not want to replace it with mathematics that emphasized systemic properties and formal structure above all else. While differences existed among the various new textbooks, he noted that "in the main they have a common curriculum" that focuses on "relearning in great logical detail the properties of numbers," wasting time that "would be better utilized in taking the students on to more advanced areas." The reforms, he warned, would do more harm than good, because "all of the desirable changes amount to no more than a minor modification of the existing curriculum, and all talk about modern society's requiring a totally new kind of mathematics is sheer nonsense."[41]

[40] H. P. Greenspan, "Applied Mathematics as a Science," *Amer. Math. Mon.*, 1961, *68*:872–880; Dahan Dalmedio, "L'essor des mathématiques appliquées aux États-Unis" (cit. n. 4), esp. pp. 189–198; Carrier *et al.*, "Applied Mathematics," pp. 297–301, 310–311; and Peter Lax, "The Flowering of Applied Mathematics in America," in *Century of Mathematics in America*, ed. Duren (cit. n. 24), Vol. 2, pp. 455–466. These trends and tensions are evident in Committee on Support of Research in the Mathematical Sciences, *The Mathematical Sciences: A Report* (Washington, D.C.: National Academy of Sciences, 1968).

[41] Morris Kline, "Math Teaching Reforms Assailed as Peril to U.S. Scientific Progress," *New York University*

Kline's critique received its most public airing in a letter signed by dozens of mathematicians and published in 1962. Even though no author was listed, it was an open secret that Kline led the effort, alongside his NYU colleague Lipman Bers and Stanford's Pólya and Max Schiffer. The letter put forward a set of principles for new curricula, three of which dealt specifically with the relationship between the nature of mathematics and instruction in the subject. First, the letter postulated that "knowing is doing" and that "knowing how" should be emphasized in the classroom. Students who understand concepts but cannot calculate or manipulate symbols, the authors claimed, do not know mathematics. Math's relevance for students emerges from concrete situations, not formalisms. Second, concepts should be introduced by examination of cases and then connected to scientific applications of mathematics. The significance of mathematics lay in the extent and success of its applications, not in its internal structure or consistency. Third, the "traditional" subjects—algebra, geometry, trigonometry, and so forth—that were "fundamental" fifty or a hundred years ago were still foundational. The problem with the traditional curriculum was not an overemphasis on these subjects but, rather, that they were taught in isolation from other domains of knowledge, especially the physical sciences, and thus came across as isolated tricks or rote manipulations rather than as a coherent body of knowledge. Coherence came from context, not from within. Noting that they could not "enter here into detailed analysis of the proposed new curricula," the authors nonetheless concluded that there were certain aspects "with which we cannot agree." The letter was not critical of any specific curriculum, but few were fooled; when the *New York Times* later covered the controversy, its reporter correctly noted that the letter was read as growing out of Kline's criticism of SMSG.[42]

SMSG's leader published his own response a few months later. Begle deflected the criticism by noting that nearly all the principles mentioned by the letter writers were also espoused by SMSG authors. (What did not need to be said was that the sites of this exchange—the *American Mathematical Monthly* and the *Mathematics Teacher*—were the organs of the Mathematical Association of America and the National Council of Teachers of Mathematics, two of SMSG's sponsors.) In subsequent inquiries, Begle found few of the mathematicians who signed the letter to have any knowledge of specific curricular materials. Many of those who did sign either believed the letter to apply only to non-SMSG textbooks or supported the letter as an ironic gesture, indicating their belief that the principles were in fact consistent with SMSG's work. Begle ultimately condemned the letter as unhelpful, in that such comments without corresponding reform efforts would never have any positive impact. While Begle's response emphasized that the letter should not be taken to imply that some large group of professional mathematicians rejected SMSG's textbooks, the whole episode revealed fissures within the professional community. One of the reputed authors of the letter, Lipman Bers, had been worried for years that SMSG might appear as an "artificially united front" that minimized very real

Alumni News, Oct. 1961, pp. 1, 3, 8; see also the compilation of his views in Kline, *Why Johnny Can't Add: The Failure of the New Math* (New York: St. Martin's, 1973). Kline had been a critic of mathematics education even prior to SMSG; see, e.g., Kline, "Mathematics Texts and Teachers: A Tirade," *Math. Teach.*, 1956, *49*:162–172. A less consequential but equally vehement critique was that of Joong J. Fang, *Numbers Racket: The Aftermath of the "New Math"* (Port Washington, N.Y.: Kennikat, 1968).

[42] "On the Mathematics Curriculum of the High School," *Amer. Math. Mon.*, 1962, *69*:189–193, on p. 191; and Robert C. Toth, "Teaching of the 'New Math' Stirs Wide Debate among Teachers," *New York Times*, 21 Sept. 1962, p. 31. See also David Lindsay Roberts, "The BKPS Letter of 1962: The History of a 'New Math' Episode," *Notices of the AMS*, 2004, *51*:1062–1063.

differences of opinion among mathematicians.[43] Even if relatively few mathematicians were as deeply engaged with curriculum as Begle, Stone, and Kline, it was understood to be consequential for the profession as a whole.

The letter emphasized two additional pedagogical claims that Kline had long been pushing and that cut to the heart of the controversy. The first was the assertion that pedagogy ought to follow the "genetic" principle. Mathematicians may care about structure and logic, but students wouldn't. In their place, Kline proposed organizing the curriculum genealogically. The 1961 letter described this by claiming, "The best way to guide the mental development of the individual is to let him retrace the mental development of the race—retrace its great lines, of course, and not the thousand errors of detail." As René Thom, a French mathematician critical of Bourbaki and SMSG, later clarified, "Pedagogy must strive to recreate (according to [Ernst] Haeckel's law of recapitulation—ontogenesis recapitulates phylogenesis) the fundamental experiences which, from the dawn of historic time, have given rise to mathematical entities."[44] The role of history was explicit, and Kline's endorsement of the "genetic" principle was founded in part on his extensive historical research. Kline's surveys of the history of mathematics laid the foundation for the claim that historical order mattered—presenting logical structures as the actual nature of mathematics did little justice to centuries of mathematical development.[45]

The "genetic" view was the only principle expressed in the letter that Begle rejected entirely. He countered by explaining the absurdity of forcing students to compute with all the historical number systems before learning the "far more efficient" system of decimals. Similarly, Stone complained that "reliving the experiences of the race" was an "old chestnut" and an incoherent precept, given the divergent intellectual trajectories of mathematics.[46] For Begle, Stone, and other SMSG partisans, mathematics in textbooks should resemble mathematics in contemporary practice, and this practice was ultimately about investigating the nature of structures. Modern mathematics had obviated the need to waste time tracing historical developments when an understanding of the underlying structure of mathematics provided students with the essential reasoning skills.

The letter also drew on Kline's long-standing opposition to the claim that mathematical structures were indeed at the heart of mathematical practice. He charged that the newer curricula did not communicate the nature of mathematics as a "living, vital, and highly significant subject," with the result that the curricula distorted the "primary value of mathematics," which was as "the language and essential instrument of science." Kline had done his doctoral work on topology, but after he moved to NYU in the 1930s Courant had convinced him that "the greatest contribution mathematicians had made and should continue to make was to help man understand the world about him."[47] For Kline, groups

[43] E. G. Begle, "Remarks on the Memorandum 'On the Mathematics Curriculum of the High School,'" *Amer. Math. Mon.*, 1962, *69*:425–426. See also Roberts, "BKPS Letter of 1962"; Folder "Re: Bers–Kline . . .," Box 13.5/86-28/53, SMSGR; and Lipman Bers to Begle, 28 Dec. 1959, Folder "Advisory Committee," Box 4.1/86-28/1, SMSGR.

[44] "On the Mathematics Curriculum of the High School" (cit. n. 42), p. 190; and René Thom, "Modern Mathematics: Does It Exist?" in *Developments in Mathematical Education*, ed. Howson (cit. n. 20), pp. 194–209, on p. 206.

[45] This research culminated in Morris Kline, *Mathematical Thought from Ancient to Modern Times* (New York: Oxford Univ. Press, 1972).

[46] Begle, "Remarks on the Memorandum 'On the Mathematics Curriculum of the High School'" (cit. n. 43), p. 426; and Marshall Stone to Begle, 20 Nov. 1961, Folder "Re: Bers–Kline . . .," Box 13.5/86-28/53, SMSGR.

[47] Kline, "Math Teaching Reforms Assailed as Peril to U.S. Scientific Progress" (cit. n. 41), p. 8; and G. L.

like Bourbaki—far from their stated goal of providing an entirely new way to conceptualize the discipline—were a passing fad. An emphasis on logical, abstract structures misconstrued the history of mathematics and the way most mathematicians actually go about their work.

The interconnection of the history and the nature of mathematics came together in the metaphor of mathematical development Kline supplied to rival Bourbaki's analogy of urban renewal. He pointed to how math is like a "great tree ever putting forth new branches and new leaves but nevertheless having the same firm trunk of established knowledge. The trunk is essential to the support of the life of the entire tree." Such images were in fact far more common than those portraying math's development as destructive. Kline himself claimed that he took the metaphor from the work of the late nineteenth-century mathematician Felix Klein, and just a couple years earlier Glenn James, the managing editor of *Mathematics Magazine* and a professor at UCLA, had edited a book called *The Tree of Mathematics*. The frontispiece and cover displayed a tree with roots labeled "algebra" and "plane geometry," a trunk of "calculus," and branches including "topology," "calculus of variations," and "probability." The organic metaphor emphasized both the permanence and the cumulativeness of mathematical knowledge and the dependence of new research on the more fundamental areas of inquiry.[48]

As with the Society for Industrial and Applied Mathematics discussion, Kline's letter and criticisms drew attention to the way SMSG's textbooks provided a venue for a larger disciplinary debate about the nature of mathematical knowledge. By the start of the 1970s, G. Baley Price, a former head of the Mathematical Association of America and the first chairman of the Conference Board of the Mathematical Sciences, looked back on the previous decade and concluded that "in recent times the mathematicians have tended to divide into two groups—the admirers of the queen and the admirers of the handmaiden." Price's quip referred to Carl Friedrich Gauss's characterization of mathematics as "queen of the sciences" and E. T. Bell's counter that it was also "handmaiden" to them. Price neatly divided the mathematical world into two camps, one that envisioned mathematics as fundamentally divorced from physical applications and another that believed that physical applications determined the course of mathematics.[49] While this was clearly an oversimplification, he was right to point out that the very nature of mathematical practice was at stake. One SMSG supporter, the mathematician Albert Meder, had wondered back in 1958, at the start of the controversy, "whether in point of fact, Professor Kline really likes mathematics." He continued, "I do not deny that he is a mathematician, and one of considerable competence. I do not deny that he is a teacher of mathematics, and from the views he expresses concerning mathematical pedagogy, I am inclined to believe that he is a good teacher of mathematics. But I think that he is at heart a physicist, or perhaps a 'natural philosopher,' not a mathematician." Meder's musings centered on the relationship between pedagogical criticism and disciplinary allegiances—allowing that one might do

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Alexanderson, "An Interview with Morris Kline: Part 2," *Two-Year College Mathematics Journal*, 1979, *10*:259–264, on p. 262.

[48] Kline, *Why Johnny Can't Add* (cit. n. 41), pp. 91–92; Morris Kline, "Logic versus Pedagogy," *Amer. Math. Mon.*, 1970, *77*:264–282, on p. 279; and Glenn James, ed., *The Tree of Mathematics* (Pacoima, Calif.: Digest, 1957). The "tree of mathematics," of course, builds on a long-standing use of tree-like structures to represent divisions of knowledge; see, e.g., John E. Murdoch, *Album of Science: Antiquity and the Middle Ages* (New York: Scribner's, 1984), esp. pp. 38–51.

[49] G. Baley Price, "Mathematics and Education," in *The Encyclopedia of Education*, ed. Lee C. Deighton, Vol. 6 (New York: Macmillan, 1971), pp. 81–90, on p. 86; for the comparison of queen and handmaiden see Eric Temple Bell, *Mathematics: Queen and Servant of Science* (New York: McGraw-Hill, 1951).

mathematics without being a *bona fide* mathematician. For his part, Kline might have replied that the mathematicians writing curricula had themselves come from outside the mathematics mainstream and were in fact—as he put it in 1961—"men from the more remote and less scientifically motivated domains of mathematics who have felt free to devote themselves to curriculum work."[50] Such attacks bear witness to how SMSG's curriculum revealed the tensions at the heart of the profession.

CONCLUSION

It was a common midcentury trope, among politicians and pundits who supported the use of federal funds for curriculum development, to exclaim about the need for more "mathematical minds." NSF's William Morrell was quoted in *Newsweek* in 1965 as claiming that "there is an intellectual content to mathematics that the educated man should have and want to have if it is presented in an interesting and exciting way. Not 'Here is a problem, solve it.' But rather, 'Here is a situation, think about it.'" *Time* magazine likewise proclaimed the year before that "math is not only vital in a day of computers, automation, game theory, quality control and linear programming; it is now also a liberal art, a logic for solving social as well as scientific problems." The new math's role in promoting these intellectual virtues enjoyed broad bipartisan support, with California's conservative head of education Max Rafferty approving it on the basis that "the old, comfortable ways are no longer enough" and President Lyndon Johnson pointing to the new math as evidence for the role that federal education initiatives might play in forging a "Great Society."[51] Conservatives were happy that mathematicians, not educators, were in charge, and liberals were pleased that the federal government was supporting educational reform. SMSG was funded and its textbooks were sold on precisely the claim that aligning practice and pedagogy in mathematics was good for American children.

These arguments proved largely successful during the early to mid 1960s, when a coalition of administrators, parents, and teachers encouraged the adoption of new textbooks. While there were some substantial concerns about the preparation of teachers, especially elementary school teachers, for presenting the unfamiliar material, the curriculum was widely adopted. Front-page articles in the *New York Times* and dozens of books preparing parents for the introduction of "modern" mathematics promoted the new math as essential to the intellectual training of students. One teacher encouraged his colleagues to follow the mathematicians' recommendations for change by reminding them that "the age of technology is upon us and we must face the changes which accompany it. The young people of today must be prepared to take a role in tomorrow's world." Another schoolteacher similarly found the new curriculum compelling, even if unfamiliar: the "modern teacher, to stay modern, must adopt modern methods and take a fresh, new outlook on methods and approaches." In the mid 1960s, the new math was heralded as a welcome and promising reform.[52]

[50] Albert E. Meder, Jr., "The Ancients versus the Moderns—A Reply," *Math. Teach.*, 1958, *51*:428–433, on p. 433; and Kline, "Math Teaching Reforms Assailed as Peril to U.S. Scientific Progress" (cit. n. 41), p. 8.

[51] "The New Math: Does it Really Add Up?" *Newsweek*, 10 May 1965, pp. 112–117, on p. 116; "Inside Numbers," *Time*, 31 Jan. 1964, pp. 34–35, on p. 35; Max Rafferty, *On Education* (New York: Devin-Adair, 1968), p. 77; and U.S. Congress, Senate, Committee on Labor and Public Welfare, Subcommittee on Education, *Elementary and Secondary Education Act of 1965: Background Material with Related Presidential Recommendations*, 89th Cong., 1st sess. (25 Jan. 1965), pp. 12–14, 16 (Johnson).

[52] Schwartz, "New Math Is Replacing Third 'R'" (cit. n. 7); Donald F. Define, "Mathematics: Formulation of

Less than a decade later, however, the new math faced widespread condemnation. Teachers questioned the way the curriculum had been forced into classrooms so rapidly, often noting that they had reverted to traditional techniques even after officially adopting the new textbooks. Many parents and teachers—especially those who became involved only after the extensive outreach and educational initiatives that accompanied the "roll-out" of new math in the mid 1960s—neither understood nor cared about the mathematical structures emphasized in textbooks. A new movement, "back to basics," flourished in the 1970s amidst claims that declining test scores provided definitive evidence that the new math had overemphasized structure at the expense of calculation ability. While the decline in scores was hardly attributable to groups like SMSG, the critics succeeded in labeling the new math a failure.[53]

Long before teachers and parents condemned the new math, a group of mathematicians had spoken out against the reforms. The problem was that it simply was not clear, even to mathematicians, how a curriculum ought to portray the discipline. SMSG did not just hope to replace rote learning with "understanding," substituting one pedagogical technique for another in pursuit of better calculation skills. Its organizers wanted students of mathematics to understand the field's various practices as exemplars of systematic reasoning, not simply as a dusty collection of facts and techniques. In modern mathematics textbooks, structure trumped content. SMSG's critics, however, claimed that the new math was pedagogically and intellectually unsound, emphasizing the faddish Bourbaki program over the more central historical developments of mathematics as the instrument of science.

This debate, at least for mathematicians, was never about the importance of mathematics or the vices of rote learning. Everyone agreed that students should learn mathematics—and learn it as something more than a stultifying set of facts. Rather, it was about whether elucidating and understanding mathematical structures could or should be taken as emblematic of mathematics in general. Even during a period of nearly unprecedented faith in the benefits of cultivating mathematical ways of thought, the underlying nature of mathematical methods was fiercely contested.

the Curriculum at Rich Township High School," *Clearing House*, 1962, *36*:460–463, on p. 460; and B. R. Reardon, "I'm for the Modern Math: Here's Why," *Alabama School Journal*, Jan. 1966, *83*:9. The genre of new math primers for parents was extensive. See, e.g., John L. Creswell, "Mom and Dad Study the New Math," *Parents' Magazine*, Sept. 1966, *41*:69, 96–98; and Darrell Huff, "Understanding the New Math?" *Harper's Magazine*, Sept. 1965, *231*:134–137.

[53] The broader analysis of the new math's reception and rejection is told in Phillips, *New Math* (cit. n. 6), esp. Chs. 5–6.